Computability and Computational Complexity, A.Y. 2024–2025

Written test

Monday, August 26, 2025

Exercise 1

For each of the following properties of a Turing machine $\mathcal{M}$, prove whether it is computable or not. When possible, invoke Rice's Theorem.

1.1) $\mathcal{M}$ accepts at least one input string.

1.2) $\mathcal{M}$ accepts at least one input string of length 3.

1.3) $\mathcal{M}$ accepts at least one input string of length 3 within 1000 steps.

1.4) $\mathcal{M}$ accepts at least one input string within 1000 steps.

Exercise 2

Prove the following assertions by describing the appropriate polynomial-time reductions:

2.1) CLIQUE $\leq_{\mathrm{P}}$ INDEPENDENT SET;

2.2) INDEPENDENT SET $\leq_{\mathrm{P}}$ VERTEX COVER;

2.3) SATISFIABILITY $\leq_{\mathrm{P}}$ 3-SATISFIABILITY.

Exercise 3

3.1) Define the classes **L**, **NL**, **P** and **NP**.

3.2) Prove that $\mathbf{L} \subseteq \mathbf{P}$.