Computability and Computational Complexity, A.Y. 2019–2020

Written test

Friday, July 3, 2020

Exercise 1

1.1) Let L be a language on a finite alphabet, and let $\mathcal{N}$ be a non-deterministic Turing machine on the same alphabet with the following properties:

- $\mathcal{N}(x)$ takes at most $|x|^2$ non-deterministic steps before halting ($|x|$ is the size of x).
- If $x \notin L$, then $\mathcal{N}(x)$ rejects the input.
- If $x \in L$, then $\mathcal{N}(x)$ accepts the input.

Are these properties sufficient for us to say that $L \in \mathbf{NP}$?

1.2) Suppose that $\mathcal{N}$ has the following additional property:

- At every step, $\mathcal{N}$ performs at most one binary non-deterministic choice (i.e., $\mathcal{N}$ has two transition functions)

Given this property and those listed in the previous point, determine an upper bound for the number $C_{\mathcal{N}}(x)$ of non-deterministic computations performed by $\mathcal{N}$ on input x as a function of the input size $|x|$.

1.3) Suppose that $\mathcal{N}$ has the following additional property:

- If $x \in L$, out of the $C_{\mathcal{N}}(x)$ computations of $\mathcal{N}(x)$, at least $\sqrt{C_{\mathcal{N}}(x)}$ end in an accepting state.

Is this additional property (and the previous ones) sufficient for us to say that $L \in \mathbf{RP}$?

Exercise 2

For each of the following properties of Turing machines, prove whether it is recursive or not. Whenever possible, use Rice's theorem.

2.1) M changes state at least once when executed on the empty input

2.2) M never remains on the same state for two consecutive steps

2.3) M accepts all inputs