

Written exam

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Wednesday, January 15, 2020

Exercise 1

Consider the UNIVERSITY HIRING decision problem:

A university needs to hire the teaching staff for a new degree, for which a set T of topics must be taught. The executive board received n applications from prospective teachers, and every applicant $i \in \{1, \dots, n\}$ has knowledge of a subset $S_i \subseteq T$ of the required topics. The budget allows for hiring at most $k \leq n$ teachers. Is there a choice of k applicants so that all teaching topics are covered?

An instance of the problem consists of parameters n, k, T, S_1, \dots, S_n .

1.1) Prove that UNIVERSITY HIRING \in NP.

1.2) Prove by reduction that UNIVERSITY HIRING is complete in the class NP. The reduction can refer to any language discussed during the course.

1.3) Prove that if k is kept constant (e.g., $k = 10$), then the problem's asymptotic complexity is polynomial wrt input size.

Exercise 2

2.1) When is a language recursive? When is it recursively enumerable?

2.2) Prove that the following property \mathcal{P} of Turing machines \mathcal{M} is not recursive:

$$\mathcal{P} = \{\mathcal{M} : \mathcal{M}(\varepsilon) \text{ halts after an even number of steps}\}$$

where ε is the empty input string.

2.3) Prove that \mathcal{P} is recursively enumerable.

Hint — *Point 2.3 can be proved by explicitly outlining an enumeration algorithm.*

Exercise 3

Consider the following language:

$$S = \left\{ (x \in \{0, 1\}^*, k \in \mathbb{N}) : x \text{ contains a subsequence of } k \text{ adjacent 0's} \right\}.$$

For example, $(101000101, 3) \in S$ because the binary string contains 3 consecutive zeroes, while $(101000101, 4) \notin S$ because the binary string does not contain 4 consecutive zeroes.

3.1) Prove that $S \in \mathbf{P}$.

3.2) Prove that $S \in \mathbf{L}$.

Hint — *Again, both points can be proved by describing an algorithm and showing that it has the required property.*